

Tree Planting Decisions Under Conditions of Irreversibility And Imperfect Labor Markets

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JANUARY, 2003

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EXECUTIVE SUMMARY

This study is based on two observations about farmers' decisions to plant trees. The first is that it takes more labor to cut or uproot trees than to plant them. The second, is that rural labor markets are riddled with imperfections that make it difficult to hire labor for farm management so households tend to rely on family labor. The combination of these two factors may lead to a situation of economic irreversibility. This eventually locks land to trees. Farm households lose their ability to cope with risk ex-post because they have less flexible farming systems. The premise of this study is that households take these factors into account when choosing the rate, timing, and scale by which they plant trees.

To verify this premise, a general model of natural resource accumulation under uncertainty and economic irreversibility was constructed. This model is a discrete time version of the common Dixit and Pindyck model. Stylized assumptions were used to conduct a numerical analysis. The objective of the numerical simulation is to derive general theoretical results and propositions about how economic irreversibility affects tree-planting decisions.

Results of the analysis indicate that labor market imperfections lead to a lower volatility in the wage process. As seen from the stylized numerical simulation, this leads to earlier planting. Therefore, contrary to the findings of the standard model of exogenous irreversibility, the possibility of delayed timing is not true for economic irreversibility. The scale effects of irreversibility are also clearly seen from the analysis. Labor deficit or more labor-constrained households tend to plant early but will plant fewer trees per hectare. Numerical simulations showed that labor deficit households would plant faster compared to labor deficit households under economic irreversibility.

It was also found that demographic variables (e.g. family size) affected the wage process and, therefore, the results can be attributed partly to these variables. Finally, the policy implication of this study is discussed. To be effective, agroforestry programs in the uplands must recognize heterogeneity of upland populations. As a consequence, the project rather than program approach is more appropriate when introducing upland interventions. These interventions must strongly consider the demographic characteristics and the position of the farming households in the labor market.

1.0. Introduction: Context and General Objective

Forest clearing for upland agriculture has long been recognized as a major cause of deforestation and land degradation in developing countries. In response, the governments of these countries have promoted agroforestry to attain both environmental protection and economic productivity. This technology was conceptualized in the late 1970's as a land use approach that is socially acceptable and can ensure sustainability of the resource base. Agroforestry since then has gained popularity and support from both bilateral and multilateral funding agencies. Millions of dollars in aid, loans, and grants are given annually to the governments of developing countries in support of agroforestry projects.

Agroforestry is defined as “a collective name for land-use systems and technologies where woody perennials are deliberately used on the same land-management units as agricultural crops and/or animal, in some form of spatial arrangement or temporal sequence and where there are both ecological and economic interactions between the different components” (Nair, 1990 quoting ICRAF). It is often said that agroforestry have numerous benefits for upland farmers. An important benefit is its promise as an effective soil fertility management technique that lead to increased farm income. This is achieved through the complex biological interactions between trees and the soil. Trees add organic matter and nutrients and promote desirable bio-physical characteristics of the soil. Their roots and litter are also good soil retainers that can reduce soil erosion. Tree roots also help in nutrient cycling. Nutrient cycling involves the turnover of nutrients already in the soil; this includes the translocation of nutrients from soil layers that are beyond the reach of annual crops (Nair, 1990). Livestock, on the other hand, can be a cheap source of organic fertilizer aside from being used as work animals.

At a regional and landscape levels, widespread adoption of agroforestry leads to a reduction in off-site erosion costs. This in turn reduces the upstream-downstream community tensions. Lowlanders (downstream communities) have often complained that soil eroded from upland farms (upstream communities) cause siltation of irrigation canals that lead to lower farm outputs. However, coordinated adoption of agroforestry systems can be harder because this involves collective action. Nevertheless, this shows that agroforestry has social benefits and the potential to solve transboundary environmental problems.

However, despite the private and social reasons from practicing agroforestry, and the large extent of agroforestry programs, farmer's adoption and investments in agroforestry has been slow. The litany of program experiences and observations from Asia, Latin America and Africa validate this claim (Scherr, 1995). This is also an indicator that there may be problems with this technology from the farmer's perspective and that policy makers have possibly neglected some important aspects of the technology adoption problem. To realize the benefits (private and social) from agroforestry, it is therefore essential to study farmer's adoption or investment behavior with respect to this technology.

Some researches have been done on the factors that encourage tree planting (Shiveley 1997, Delos Angeles, 1979, Govil, 1994, and Scherr et. al, 1995). These studies usually replicate what is often found in studies of technology adoption of agricultural crops. The empirical method of choice is either logit or probit estimation of reduced form equations. Here factors such as labor, credit, and informational constraints are treated as independent variables that affect the probability of adoption or planting trees. These studies use cross-section data thereby making an implicit static treatment of the adoption process. However, tree planting and agroforestry decisions are inherently dynamic processes. In fact, they are better viewed from an investment rather than an adoption perspective since farmers pay upfront input costs and receive benefits at a later period.

This study adopts the investment view as basis for agroforestry land-use decision. Within this perspective, it will focus on the issue of labor market failure and its implications on a type of silvicultural agroforestry system ---- fruit tree-crop combination. This subject has been dealt with indirectly in earlier studies by analyzing the impact of labor constraints on the probability of tree planting. What was overlooked is how this socio-economic condition interacts with the biophysical nature of trees. Trees or perennials unlike rice, corn or other annual agricultural crops, have long gestation periods. Once they reach a certain stage of maturity, they become sturdy and deep-rooted. At this stage, removing and replacing them with other crops require a significant amount of resources and effort. In contrast, annual agricultural crops can be plowed under the soil and are easily replaceable with other crops. Often farmers would leave annual crops to dry and then they plow them under the soil to be used as fertilizers for the replacement crops. The same technique cannot be applied to trees because their deep roots, which enable them to withstand long dry periods, become great obstacles for immediate land restoration for crop growing. This becomes a problem when trees cease to be a viable income source during periods when the household is labor constrained because it makes it more difficult to restore land back to suit a different cropping pattern. In short, trees become irreversible investments. This in turn reduces the household's flexibility and their ability to respond

to changing economic conditions. Since changing cropping patterns is a way to deal with risk, planting trees has the potential to curtail the household's ability to cope with risk, ex-post. The hypothesis is that this interaction is accounted for by farmers and, therefore, reflected in their decision-making with regards to the timing, rate, and intensity of crop-fruit tree combinations.

Labor market imperfections also have implications for the theory of investment under irreversibility and uncertainty. Investment under irreversibility and uncertainty has received a great deal of attention in recent economic literatures (Dixit and Pindyck (1994), Pindyck (1985), Abel (1993)). To study their effect on investment, researchers have turned to concepts from finance in particular, as they relate to analogies from options trading. This emerging body of investment literature has been referred to as the Real Option Theory (ROT) of investment (Dixit and Pindyck, 1994). The centerpiece of this investment theory is the value of waiting for more information and the resulting delay in investment. It is shown that investment hurdles are higher than that proposed by the usual benefit-cost analysis because information and the need for flexibility entail an option value that favors the status quo. Models are often framed as an optimal stopping problem, thus they tend to describe the discrete choice of whether to invest or not at each point in time.

A number of studies also deal with continuous choice. The interest on continuous choice models is characterizing both the timing and intensity of investment. Despite small differences in model structures, the ROT relies on three major considerations; (a) investments must be partially or completely irreversible, (b) there is uncertainty over future rewards or costs of investment, and (c) decision makers have some leeway about the timing of the investment (Dixit and Pindyck, 1994). The uncertainty in prices or costs is embodied in the assumptions about the price or cost process. The standard assumption is that of a Brownian process for either the prices or the costs and the parameters of these processes are assumed to be determined exogenously.

The existence of irreversibility is often tied up with the concept of sunk costs that reflect the difference between the initial investment and restoration costs. Irreversibility becomes an apparent concern if the restoration costs are significantly greater than the investment costs. This leads to an unfulfilled level of disinvestment and therefore losses to an investor. The ROT literature describe situations where technology for revising and/or replacing an investment is non-existent or is existent at a cost close to infinity (McDonald and Siegel (1985), Dixit and Pindyck (1994), and Abel and Eberly (1993)). This form of irreversibility is called technical irreversibility and often leads to an implicit assumption of complete irreversibility. In other words, irreversibility is considered exogenous to the model.

Restoration of land allocated to trees however, is always technically feasible. Technical irreversibility is a very limited concept in the context of agroforestry decision-making. What is a more relevant form of irreversibility for agrarian or upland environment is economic irreversibility. Economic irreversibility is a situation wherein it is technically feasible to reverse an action but it may not be optimal to do so (i.e. costly reversibility). Irreversibility is not exogenous as is commonly assumed in the ROT literature but rather a part of the decision process. There may be varying degrees of irreversibility including partial irreversibility. The extent of irreversibility then becomes farmer specific and is affected by the degree of the labor constraint or the severity of the market failure.

In a situation characterized by a binding labor constraint, the wage the household faces becomes endogenous. The wage by which they base their labor decisions is determined within the household and simultaneously chosen with allocation decisions. The equality between farm production labor needs and the household labor supply determine this shadow wage. It is this shadow wage that determines allocation and not the market wage, since market prices no longer reflect the full opportunity of labor. If we are to assume that uncertainty lies in the wage process, then the stochastic process underlying it can no longer be treated exogenously. The shadow wage is now an endogenous variable determined through the equilibrium of the household labor supply and demand. Unlike the static agricultural household model, it not only the wage level that must be considered but also the parameters of the stochastic process that clears this internal household market at each instance of time. Instead of finding a fixed point, the solution involves looking for a fixed stochastic process. So far, there has been no attempt to study irreversibility under an endogenous stochastic process.

To study the implications of labor constraints and irreversibility on agricultural crop-fruit tree combinations, a discrete time-continuous state simulation model is developed in the next section. This simulation model will be run under four different scenarios. The third section describes how the model will be implemented and the parametric assumptions behind the model. The fourth section discusses the results of the simulation runs and the fifth section spells out the conclusions and the policy implications of the study.

2.0. The Model

2.1. Household Income and the Wage Process

This model considers a household who owns or has secure tenure to L hectares of land at time t . At the start of the planning horizon (or at the start of farming), the land either has been clear-

cut and is ready for upland agriculture (i.e. an open field) or is currently planted with annual crops. The household decides at each period the amount of land, l_{it} to be converted or planted with fruit trees. If they have planted at an earlier date i ($i < t$), then the household would have a current stock of l_{it} hectares of fruit trees. The difference between i and t represents the age of this stock of trees. The household could also cut or restore some land back to annual cropping. If h_{it} is the amount of hectares of trees cut or restored land, then with full land allocation, the amount of land allocated to annual crops at time t would be $L - \sum_i (l_{it} - h_{it})$.

Household income y_t is assumed to be derived from returns from annual cropping, perennial cropping and off-farm work (O_t). Gross returns from perennial and annual crops is determined as follows. Perennial crops planted at time i at period t , is assumed to produce g_{it} kilograms of fruits per hectare. Notice that this output is age dependent and $g_{i't} > g_{it}$ for $i' > i$, that is, output is assumed to increase with age¹. Households also face a price P_t^p for perennials, measured in pesos per kilogram. Therefore, the gross returns from perennial cultivation at time t can be written as:

$$P_t^p \left(\sum_{i=0}^{t-1} g_{it} (l_{it} - h_{it}) \right) + P_t^p g_{tt} l_{tt}$$

Likewise, the gross return from annual cultivation can be written as:

$$P_t^a \left(L - \sum (l_{it} - h_{it}) \right)$$

where P_t^a is the market price for annual crops and a is the constant output per hectare measured in kilograms per hectare.

In this model, the assumption is that households have full knowledge of the trajectories of P_t^a and P_t^p . The randomness or uncertainty lies in the wage process or the cost of farm production and the returns from off-farm work. The assumed wage process can be described as follows; with a base wage w_0 at the start of the planning horizon, the wage at next (and subsequent) periods can either go up by an amount u or go down by an amount d . The uptrend u and the downtrend d depends on the values of the instantaneous variance σ . These trends occur with a certain probability π . Specifically, the probability of the wage going up in the next period is denoted by π and the probability of going down is $1-\pi$. This probability is influenced both by the growth rate of the wage α , and by the

¹ Usually during the first three to four years, trees do not produce any output.

instantaneous variance. This is simply a binomial process², which is a non-stationary stochastic process. More formally, this process can be written as:

$$w_{n,t+1} = w_0 u^n d^{t-n-1} \quad (1)$$

where

$$u = \hat{\sigma}$$

$$d = \frac{1}{\hat{\sigma}}$$

$$\pi = \frac{e^{\hat{\alpha}-d}}{u-d}$$

n = no. of times that there was an increase in wages

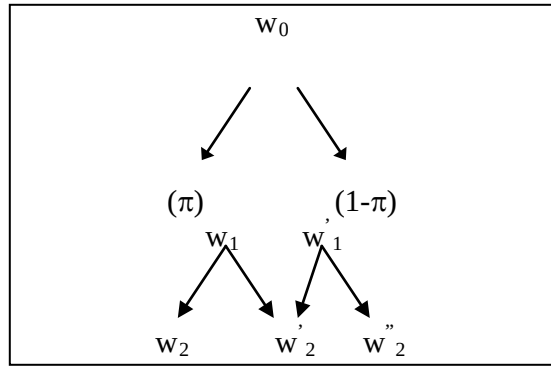


Figure 1. Binomial Wage Process

In upland agriculture, the main agricultural input is labor. To simplify the model, land and labor decisions are linked through scalar parameters representing labor requirements per hectare. For perennials, there is an assumed age (growth stage) dependent labor requirement per hectare denoted by n^p_{it} . This parameter is measured in man-days per hectare and represents planting, cultivation and harvesting requirements. The value of n^p_{it} depends on the species of trees and the cultivation practices adopted by farmers. Forest trees often require more labor at early stages of growth and a one-time harvest during maturity. On the other hand fruit bearing trees require more labor during the early stages of growth and for episodes of harvesting during maturity. In general therefore, labor requirements are high during the “weaning” periods of trees and varies during the harvesting season depending on the type of trees. Farmer’s cultivation practices, however, can affect the labor allotted to tree cultivation. Some farmers do not practice ideal tree cultivation techniques. Trees are often left

² Cox, Rubinstein, and Ross (1977) and Nelson and Ramaswamy (1990) have shown that the geometric Brownian motion is the limit of a binomial process under certain conditions. The geometric brownian process has been used extensively in the study of investment under irreversibility and uncertainty. This process is commonly used to model the behavior of economic and financial variables such as stock prices, GNP, wages, etc. (Dixit and Pindyck, 1994).

to the elements during the early stages of growth and labor is allotted only during harvesting should the trees survive up to maturity. Under this type of practice, labor is quite high during harvesting with little or no time spent during the early stages of tree growth. This is especially true for trees that grew unintentional in farmer plots.

The restoration or tree-cutting labor requirements per hectare, n_{it}^c , on the other hand are likewise age dependent. Furthermore, it is assumed that $n_{i't}^c > n_{it}^c$ for $i' > i$, that is, it requires more man-days per hectare to cut older trees. This biological property of trees increases the chance of irreversibility in the future. Irreversibility is a direct effect of the “wedge” between the planting cost (investment cost) and the cutting cost (adjustment costs).

Annual crop labor requirement is assumed to be age-independent and is, therefore, a constant. This constant labor requirement is represented by n_a . The last assumption is that household can allocate its labor in the external labor market. Off-farm work was designated as O_t and is measured in man-days.

With these assumptions the household income equation can be written as:

$$y_t = P_t^a \left(L - \sum_{i=0}^t (l_{it} - h_{it}) \right) + P_t^p \left(\sum_{i=0}^{t-1} g_{it} (l_{it} - h_{it}) \right) + P_t^p g_{tt} l_{tt} - w_t \left(\sum_{i=0}^t n_{it}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it}^c h_{it} + n_a (L - \sum_{i=0}^t (l_{it} - h_{it})) - O_t \right) \quad (2)$$

2.2. Nature of the Irreversibility Constraints

There are two possible descriptions of the nature of the irreversibility constraint. First possibility is that the constraint is exogenously set. This can happen for instance because of cultural prohibitions that limit tree-cutting activities. Government regulations can also hinder tree cutting in the future. The transaction costs associated with the government's cutting permit system can be so high that it may not be prudent for farmers to cut their trees. Whatever the reason maybe, exogenous irreversibility effectively locks land use to trees and restricts household's flexibility in land use decisions. Note that this constraint equally affects any household. Exogenously constrained tree-cutting in the future can be represented by the following constraint:

$$h_{it+1} \geq 0 \quad \text{for } i = 0, \dots, t \quad (3)$$

when $h_{it}=0$ then there is full irreversibility.

Alternatively, the irreversibility constraint can also be endogenously set. This means that it is possible to restore some land back to other use, but it is smaller than what would have been possible when there are no constraints. Unlike the previous case, irreversibility is unique to each household depending on the unique factors that face each household. This model focuses on labor availability as the factor that prevents households from achieving the desired land allocation. Imperfect markets that lead to constraints in economic activity often characterize archetypal agrarian or upland environments. The labor market³ is one such market that is prone to imperfections or failures. These market failures arise for several reasons. Informational asymmetries often lead to high transactions cost (e.g. monitoring and supervisory costs) for hired labor. Hired labor is also subject to incentive problems that may reduce their productivity and lead to higher total costs for the employer. Farmers may also have simultaneous labor needs every cropping season. This is especially true when there is very little crop diversification at the village level. These covariant needs often lead to a shallow or thin labor markets. All these examples emphasize that when there are significant transactions costs, asymmetric information or risk, labor markets may fail. Consequently, the household is forced to rely on the household labor pool, N_t to meet on farm labor demand. The constraint, therefore, represents the assumption that households are not able to hire-in labor. This constraint can be represented by the following equation:

$$N_t \geq \sum_{i=0}^t n_{it}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it}^c h_{it} + n_a (L - \sum_{i=0}^t (l_{it} - h_{it})) + O_t \quad (4a)$$

The household stock of labor N_t is determined by changes in the household composition. Several factors in turn affect household composition. First, change is brought about by out migration and in migration of household members. Second, changes occur through increase or decrease in the work capacity of existing household members. This factor tracks the aging or life cycle of the household. If migration is denoted as m_t and out migration as d_t , then the equation of motion for labor stock or household labor supply is:

$$N_t = N_{t-1} + m_t - d_t$$

If the labor constraint is binding then the equation of motion can be alternatively written as:

$$N_{t+1} = m_t - d_t + \sum_{i=0}^t n_{it}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it}^c h_{it} + n_a (L - \sum_{i=0}^t (l_{it} - h_{it})) + O_t \quad (4b)$$

³ Most studies have focused on imperfections in the credit market (Feder (1982), Smale et. al. (1996), dela Breire (1996), Eswaran and Kotwal (1987), Green and Ng'ong'ola (1993)).

A related consequence of labor market failures is that the wage rate that the household face is no longer the market wage. The wage rate is internal to the household and is determined endogenously. The endogenous or shadow wage is the wage that equilibrates the household labor supply with labor demand. Because of this, it is expected that the shadow wage will also be a function of the land and labor allocations. Therefore, the shadow wage will depend on the extent of possible land restoration and on changes in the household labor stock.

The value of the shadow wage compared with the market wage will eventually affect labor decisions. Take the case where there is a wedge in the purchase and sale price of labor due to transaction costs. If the shadow wage is higher than the purchase price of labor then the household will tend to hire in labor. On the other hand, if the shadow wage is lower than the sale price of labor then the household will become a net seller of labor. Lastly, if the virtual wage falls within the wedge, the household will remain autarkic.

In the model, it is not only the wage level that is determined endogenously (as in the static case with no uncertainty), but also the parameters of the binomial process. Note that the shadow wage can also be treated as the market wage rate marked up by the Lagrange multiplier. This mark-up is not constant and its value depends on whether the labor constraints are binding or not and how strong the constraints are binding. Because of the nature of the Lagrange multiplier, it will alter the market wage process. Therefore, the parameters of the shadow wage process are different from the market wage process. Furthermore, these parameters are determined simultaneously with the labor/land allocation decisions. For reference, the equilibrium base wage is denoted as w_0^* , the equilibrium instantaneous variance as σ^* , and the equilibrium drift as α^* .

The standard model of investment under irreversibility and uncertainty corresponds to the situation of full irreversibility and the degree of irreversibility is exogenous. This corresponds to restricting $h_{it+1}=0$ in equation 3. Full irreversibility is synonymous to imposing an infinite cost with downward adjustments in investments or disinvestments. This presents an assumption of extreme asymmetry in adjustment costs. If restoration is possible, the degree of irreversibility becomes endogenous (Zhao, 1996). This seems to be a more plausible model since extreme asymmetries in adjustment costs are rarely realistic and operational. There might be some difference between the two cases because if an investment is irreversible, there is an option value conferred to not investing. This is equivalent to the value of retaining the option to make an irreversible decision until a future time conditional on more or new information. With new information, the decision-maker avoids the possibility of downside risk. The option value often causes delay or reservations in investing.

These ideas can be translated into the model in the following way. The household knows the prices of farm products and the labor requirements of a specific farm production plan, as was assumed earlier. However, the wage realization maybe too high and should prices drop (a known event) then the household stand to loose income. As mentioned earlier the cutting costs are higher because it requires more labor. Irreversibility is a direct cause of the asymmetry between planting and cutting costs. With irreversibility in land use allocation involving trees, this would mean a loss for the farmers. Should this trend persist over long periods, the household would expose itself to greater long-term risk. As a result, households would then tend to be more cautious in planting trees and would probably opt for annual crops, which offer a more flexible land use system.

However, the possibility of restoration or the reversal of action (even partial) in the future reduces this option value since, downside risk is likewise reduced with restoration. Ex-post correction of investment levels can be a means of coping with risk, which may encourage early investment. What are working are two counteracting forces wherein the actual level and pace of investment is a result of the interaction between these two forces. What the assumption of pure or exogenous irreversibility does is that it eliminates the later incentive and, therefore, only the option value is present. The value of restoration possibilities is only present when there is endogenous irreversibility which makes this a richer and more realistic case than what is common in current literature.

2.3. *The Equation of Motion for the Stock of Trees*

As households plant tree continuously over time, the land area allotted to trees accumulates. At any point in the planning horizon they would have plots that has a specific age distribution of trees. To understand the accumulation of tree stocks and how this age distribution is created it is necessary to describe the timing of tree planting and cutting activities.

At the start of any period t the household would have a stock of l_{it} hectares of trees with age $(t-i)$. If the household plants a new tree then he would have l_{it} hectares of newly planted land. During this period, if the household decides to cut, he would do so at a rate of h_{it} hectares. Therefore, at the start of the next period the household would have l_{it+1} hectares of trees that are now aged $(t+1-i)$. In equation form the equation of motion or tree stock accumulation can be written as:

$$l_{it+1} = l_{it} - h_{it} \quad \text{for } i = 0 \dots t \quad (5)$$

Using this equation the exogenous irreversibility constraint can also be written as:

$$l_{it+1} - l_{it} \geq 0 \quad \text{for } i = 0 \dots t \quad (5a)$$

This completes the model specifications. The next section outlines some analytical insights that can be gleaned from this model.

2.4. The Programming Problem and Analytical Results

This section now puts together the various assumption outlined earlier to form the programming problem facing households engaged in tree-crop cultivation. The household's objective is to maximize the net present value of utility which is denoted by $V(\bullet, \bullet)$. They are assumed to receive utility from income (or consumption) and current income y_t is deterministic, but future income streams $y_{t+1}, y_{t+2}, \dots, y_T$ are uncertain. These elements put together results in the following programming problem:

$$\text{Problem P:} \quad V(w_t, N_t) = \max_{l_{it}, h_{it}, o_t} \sum_{t=0}^T E_t \beta^t U(y_t) \quad \text{s.t.} \quad (6)$$

$$(6a) \quad y_t = P_t^a \left(L - \sum_{i=0}^t (l_{it} - h_{it}) \right) + P_t^p \left(\sum_{i=0}^{t-1} g_{it} (l_{it} - h_{it}) \right) + P_t^p g_{tt} l_{tt} - w_t \left(\sum_{i=0}^t n_{it}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it}^c h_{it} + n_d (L - \sum_{i=0}^t (l_{it} - h_{it})) - o_t \right)$$

$$(6b) \quad l_{it} = l_{it-1} - h_{it} \quad i = 0 \dots t$$

$$(6c) \quad \begin{aligned} w_{n,t+1} &= w_0 u^t d^{n-t-1} \\ \text{where} \\ u &= \hat{\sigma} \\ d &= \frac{1}{\hat{\sigma}} \\ \pi &= \frac{e^{\hat{\alpha}-d}}{u-d} \\ n &= \text{no. of times that there was a decrease in wages} \end{aligned}$$

This is the unconstrained case where no irreversibility is being assumed and is the base case by which the exogenous and endogenous irreversibility cases will be compared. If in addition equation 3 is included, then the programming problem becomes the exogenous irreversibility model. Alternatively, replacing equation 3 with equation 4 results into the endogenous irreversibility model. In general notation, the different cases can be represented by the following recursive Bellman equation:

$$V(l_{it}, w_t) = \max_{l_{it}, h_{it}, O_t \in \Theta} U(y_t) + \beta[(\pi)V(l_{it+1}, w_{t+1}) + (1 - \pi)V(l_{it+1}, w_{t+1})]$$

where Θ represents the relevant constraint equations for each case. The structure of the recursive equation follows from the assumed binomial wage process. The second period choice therefore, revolves around whether to invest (plant), reduce land allocation to trees (disinvest), or do nothing (leave existing land allocation). Note however, that the decision variables are continuous variables. These decisions are done in an environment where labor constraints are either binding or not. Notice also that the labor constraints are present only at the next period and not during the current period. This emphasizes that irreversibility can occur only in a future and uncertain date.

Solving the different models results into the following analytical results. The first and simplest case is where there are no constraints that are binding. The necessary condition for this case is as follows:

$$\begin{aligned} \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial l_{it}} + \beta(\pi(\frac{\partial V_{t+1}}{\partial l_{it+1}} \frac{\partial l_{it+1}}{\partial l_{it}}) + (1 - \pi)(\frac{\partial V_{t+1}}{\partial l_{it+1}} \frac{\partial l_{it+1}}{\partial l_{it}})) &= 0 \\ \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} + \beta(\pi(\frac{\partial V_{t+1}}{\partial h_{it+1}} \frac{\partial h_{it+1}}{\partial h_{it}}) + (1 - \pi)(\frac{\partial V_{t+1}}{\partial h_{it+1}} \frac{\partial h_{it+1}}{\partial h_{it}})) &= 0 \\ i &= 0, \dots, t \\ \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial O_t} &= 0 \end{aligned}$$

These can be simplified into the following first order conditions through the envelope theorem:

$$(7a) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial l_{it}} + \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} = 0$$

$$(7b) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} + \beta[\pi(-\frac{\partial U_{t+1}}{\partial l_{it+1}} - \frac{\partial U_{t+1}}{\partial h_{it+1}}) + (1 - \pi)(-\frac{\partial U_{t+1}}{\partial l_{it+1}} - \frac{\partial U_{t+1}}{\partial h_{it+1}})] = 0 \text{ for } i = 0 \dots t - 1$$

$$(7c) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} = 0$$

Equation 7a shows that under perfect markets tree planting is determined solely by current conditions where only the current stock of trees is relevant to both tree cutting and planting. Furthermore, there is separability in the households labor allocations.

If the labor constraint is binding then the first order conditions are:

$$\frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial l_t} + \beta E_t \left(\pi \left(\frac{\partial V_{t+1}}{\partial l_{t+1}} \frac{\partial l_{t+1}}{\partial l_t} \right) + (1 - \pi) \left(\frac{\partial V_{t+1}}{\partial l_{t+1}} \frac{\partial l_{t+1}}{\partial l_t} \right) \right) + \beta \lambda_{t+1} \frac{\partial N_{t+1}}{\partial l_t} = 0$$

$$\frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} + \beta E_t \left(\pi \left(\frac{\partial V_{t+1}}{\partial l_{t+1}} \frac{\partial l_{t+1}}{\partial h_{it}} \right) + (1 - \pi) \left(\frac{\partial V_{t+1}}{\partial l_{t+1}} \frac{\partial l_{t+1}}{\partial h_{it}} \right) \right) + \beta \lambda_{t+1} \frac{\partial N_{t+1}}{\partial h_{it}} = 0$$

$i = 0, \dots, t$

$$\frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial O_t} \beta \lambda_{t+1} \frac{\partial N_{t+1}}{\partial O_t} = 0$$

$$N_{t+1} = m_t - d_t + \sum_{i=0}^t n_{it+1}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it+1}^c h_{it+1} +$$

$$n_a \left(L - \sum_{i=0}^t (l_{it} - h_{it}) \right) + O_{t+1}$$

$$N_t \geq \sum_{i=0}^t n_{it}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it}^c h_{it} + n_a \left(L - \sum_{i=0}^t (l_{it} - h_{it}) \right) + O$$

$$\lambda_{t+1} N_t = 0$$

$$\lambda_{t+1} \geq 0$$

Through the envelope theorem these equations can be simplified into the following more useful form.

$$(8a) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial l_{it}} + \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} - \left(\frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial O_t} \right) n_{tt+1}^c = 0$$

$$(8b) \quad \begin{aligned} & \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} + \left(\frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial O_t} \right) (n_{it+1}^p - n_{it+1}^c + n_a) - \beta \pi \left(\frac{\partial U}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial l_{it+1}} + \frac{\partial U}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial h_{it+1}} - \left(\frac{\partial U}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial O_{t+1}} \right) n_{it+2}^c \right) \\ & - \beta (1 - \pi) \left(\frac{\partial U}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial l_{it+1}} + \frac{\partial U}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial h_{it+1}} - \left(\frac{\partial U}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial O_{t+1}} \right) n_{it+2}^c \right) - \beta \left(\frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial O_t} \right) n_{tt+1}^c \\ & \text{for } i = 0 \dots t-1 \end{aligned}$$

$$(8c) \quad N_{t+1} = \sum_{i=0}^t n_{it+1}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it+1}^c h_{it} + n_a (L - \sum_{i=0}^t (l_{it} - h_{it})) + O_t$$

$$(8d) \quad N_t \geq \sum_{i=0}^t n_{it}^p (l_{it} - h_{it}) + \sum_{i=0}^t n_{it}^c h_{it} + n_a (L - \sum_{i=0}^t (l_{it} - h_{it})) + O$$

$$(8e) \quad \lambda_{t+1} N_t = 0$$

$$(8f) \quad \lambda_{t+1} \geq 0$$

Several things can be noticed from this system of equations. First, unlike the perfect market case, future variables enter the decision for planting and cutting. Equation 8a shows that current planting takes into consideration the labor cost of cutting in the future. This is weighted by the marginal utility from off-farm work. On the other hand, equation 8b indicates that current cutting for existing tree stocks takes into consideration future labor costs two periods ahead. The second, thing this tells us is that now there is no separability in household land and labor allocation. Specifically, the marginal cost of off-farm work enters the marginal condition for planting and cutting. Lastly, the presence of the household labor supply in the equation means that planting decisions are now affected and partly determined by demographic characteristics of the household. In particular, it can be expected that investment patterns will be affected by the stage the household is in with respect to its life cycle.

Lastly, it is possible that the irreversibility is due to factors other than the labor constraint. When irreversibility is exogenously imposed, the relevant optimum conditions for cutting and tree planting are:

$$(9a) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial l_{it}} + \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} - \beta \lambda_{it+1} = 0$$

$$(9b) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial h_{it}} + \beta \left[\pi \left(-\frac{\partial U_{t+1}}{\partial l_{it+1}} - \frac{\partial U_{t+1}}{\partial h_{it+1}} \right) + (1-\pi) \left(-\frac{\partial U_{t+1}}{\partial l_{it+1}} - \frac{\partial U_{t+1}}{\partial h_{it+1}} \right) \right] = 0$$

for $i = 0 \dots t-1$

$$(9c) \quad \frac{\partial U}{\partial y_t} \frac{\partial y_t}{\partial O_t} = 0$$

$$(9d) \quad l_{it+1} - l_{it} \geq 0 \quad \text{for } i = 0 \dots t$$

$$(9e) \quad \lambda_{it+1} (l_{it+1} - l_{it}) = 0 \quad \text{for } i = 0 \dots t$$

$$(9f) \quad \lambda_{it+1} \geq 0 \quad \text{for } i = 0 \dots t$$

Again, several things are evident from these equations. First, is that unlike in the previous model, only current planting is affected by the constraint. This means that only planting and cutting of new trees are affected by future variables, in particular the Lagrange constraint. These results are shown by equation 9a. From 9b, the decision to cut the current stock of trees is not different from the marginal condition of the perfect market case. Unlike the previous model, there is also separability between land and labor allocation decisions. Equations 9a to 9b does not include the marginal utility derived from off-farm work unlike in the labor constraint model. Lastly, demographic variables do not enter in any way in the land and labor decisions. These last two results delineates the exogenous irreversibility from that of the endogenous irreversibility model.

Obviously, when no constraints are binding for all periods, then the problem is simply the unconstrained or the base case; that is the Euler equations for all three models are the same. Delineation of the results among the models would only be possible if at least one constraint bind for at least one period.

Though analytical results to characterize the patterns on investment or disinvestments on natural resource capital is possible, characterizing the shadow wage process would be harder. A 10-year horizon for example would have 1024 possible shadow wage combinations. A computer simulation will be used instead to gather more theoretical propositions that maybe embodied in this type of model.

3.0. Estimation of the Drift and Instantaneous Variance of the Exogenous Wage⁴

To estimate the parameters of the stochastic market wage process, a new variable $m_t = \log(w_t)$ is first introduced. Since w_t is a geometric Brownian motion, Ito's lemma can be used to obtain:

$$dm_t = \left(\alpha - \frac{1}{2}\sigma^2\right)dt + \sigma dz$$

where dz is a standard Brownian motion. Discretizing this equation will yield:

$$m_{t+1} = m_t + \left(\alpha - \frac{1}{2}\sigma^2\right)\Delta t + \sigma(z_{t+1} - z_t)$$

This discrete version follows from the assumption that $\Delta t = 1$, where yearly changes in the variables is considered. Setting $\Delta z = e$, the equation can be further simplified to:

$$e = \frac{m_{t+1} - m_t - \left(\alpha - \frac{1}{2}\sigma^2\right)}{\sigma}$$

$$e \sim N(0,1)$$

With this assumption the Log-Likelihood function for the observed market wages can be written as follows;

$$\log p(w_1, w_2, \dots, w_T) = -\frac{1}{2} \sum_{t=1}^T \left[\log(2\pi\sigma^2) + \frac{(m_{t+1} - m_t - (\alpha - \frac{\sigma^2}{2}))^2}{\sigma^2} \right] + \log p(y_0) - \sum_{t=0}^T \log w_t$$

A maximum likelihood estimation will yield estimates for α and σ .

The data for the likelihood estimation was obtained from various yearly publications of the Philippine Yearbook and the nominal wage rate for non-plantation agricultural workers were obtained from Eastern Visayas from 1973 to 1997. Table 1 shows the results of the estimation as well as the results of the simulation runs which will be discussed later.

Table 1. Exogenous Wage Process and Shadow Wage Process

Assumptions	Base Wage	Drift	Instantaneous Variance
Base Case	20.95	0	0.22
PI, LC, L=1	22.85	1.09	1.35
FI, LC, L=1	104.98	1.46	2.24
PI, LC, L=5	110.44	0.24	0.12
FI, LC, L=5	42.98	0.25	0.20

PI - Partial Irreversibility

FI - Full Irreversibility

⁴ This subsection follows the method used by Yoshimoto and Shoji (1998) in searching for the optimal rotation age for forest stands under stochastic log prices.

LC – Labor Constraint
L – Farm Size

4.0 Other Assumptions on Exogenous Parameters

4.1. Farm Size

Two values for the farm size will be used; that the household owns 5 hectares of land and the second is a farm size of 1 hectare. With these assumptions, comparison of the effect of farm size on land allocation under irreversibility and uncertainty is possible. Since, labor and land allocation decisions are linked, it is expected that labor constraints will be more binding for relatively larger farms, *caeteris paribus*.

4.2. Prices

The data for prices of annual crops were taken from various publications of the Bureau of Agricultural Statistics. Nominal prices (Pesos/kg.) for mature white corn from 1983 to 1989 for Central Visayas and 1990 to 1993 figures for Metro Cebu were used. The Bureau of Agricultural Statistics only started collecting city level and provincial level data in 1990 so that the only available data prior to 1990 was at the regional level. The choice of corn instead of rice follows from Cebuanos having strong preferences for corn over rice as their staple food, therefore, corn is more widely cultivated compared to rice in these areas.

For the perennial crop prices, mature carabao mango prices (Pesos/kg.) were used. Cebu is a major producer of export quality carabao mangoes. Again the nominal prices from 1983 to 1989 are figures for Central Visayas (regional level) and the 1990 to 1993 figures are for Metro Cebu (city level).

4.3. Household Labor Force

The simulation used man-days as measure of labor demand and supply. Man-days can vary between 5 to 8 hours of work in each day. The assumption is that the household labor force follows the life cycle stages of the family. In the early part of the family, only the husband and the wife contribute to the labor force. The husband accounts for one man-day while the wife only works 0.75 man-day in the farm. This follows from the fact that there are gender specific activities and the wife has to allocate time for household chores too.

Another critical assumption is the birth rate of the household. The birth rate assumption is that there are three children born at two-year intervals (1984, 1986 and 1988). Each child under the age of seven is a net user of labor. Specifically they contribute nothing to the total labor force and uses up 0.25 man-day of the mother's time. This means that child weaning takes away 0.25 man-days off of the mother's farm work time. A child becomes a net contributor to the labor force at the age of seven. From age seven to ten a child is assumed to contribute 0.25 man-days of work. This increases to 0.5 man-days at the age of 10 to 15, and levels at 1 man-day above 16 years of age. The abrupt change in man-days from 15 to 16 accounts for the fact that schooling for children usually ends at this age. School time accounts for a large part of a child's time allocation until their teens. The total man-days in a year is multiplied by 211 to get the total labor force for that year. 211⁵ is the number of week-days in each year.

4.4. Labor Requirements for Perennial and Annual Cropping

Corn cultivation has labor requirements of around 40.96 man-days during the dry season and 40.89 man-days during the wet season. These data are for open-pollinated or traditional varieties and include all cultivation and harvesting labor requirements in each year.

As discussed earlier, labor requirements for perennial cropping are assumed to change according to the age of the trees. Tables 2 and 3 show the cultivation and restoration requirements for mangoes in man-days/ha./year.

⁵ Some farmers usually work on weekends, in particular on Saturdays since Sundays are usually for rest. But religious festivals and other celebrations may cancel out these extra workdays, so 211 is still a safe assumption.

Table 2. Annual Cultivation Labor Requirements (man-days/ hectare)

Crop		Labor Requirement (man-days/ha.)
Corn	Wet Season	41
	Dry Season	40
Mango		
Age	1 st Year	108
	2 nd Year	37.0
	3 rd Year	37.0
	4 th Year	60.5
	5 th Year	65.5
	6 th Year	69.0
	7 th Year	74.0
	8 th Year	88.5
	9 th Year	93.5
	10 th Year	93.5

Table 3. Restoration Requirements for Mango (man-days/ hectare)

Age	Labor Requirement
1 st Year	1.25
2 nd Year	2.50
3 rd Year	3.75
4 th Year	5.00
5 th Year	6.25
6 th Year	12.50
7 th Year	12.50
8 th Year	12.50
9 th Year	12.50
10 th Year	25.00

4.5. Output/ Crop Production

For corn, there are usually two croppings per year; one for the wet season and another for the dry season. During the dry season a hectare of corn yields 3,268.47 kg. and 2,701.54 kg. during the wet season. In total, a hectare of corn can produce as much as 5969 kg. during the year. These assumptions hold for the traditional variety (open pollinated corn) which is the preferred variety of upland farmers.

The output of mango varies depending on the age of the tree. Table 4 shows the yield (in kg.) of a tree at each age. To get the yield per hectare we multiply the data by 100. This conversion relies on the assumption that farmers follow the recommended planting distance of 10 by 10 meters.

Table 4. Annual Mango and Corn Yields (kg. /ha.)

Mango		Corn
Tree Age	Yield	Yield
1	0	Wet season
2	3	2,701.54
3	5	Dry Season
4	7	3,268.47
5	10	
6	80	
7	150	
8	250	
9	400	
10	550	

4.6. Consumption and Preference

It is assumed that the unitary household has the following utility function:

$$U_t = \frac{y_t^{1-\theta}}{1-\theta}$$

Initially, θ is set at 0.4. The results from this assumption will be compared with θ set at 1.4. The instantaneous utility function is assumed to be of the isoelastic form. The basic property of this functional form is that the elasticity of substitution of income between two periods is constant and is

equal to $1/\theta$. θ also acts as the coefficient of relative risk aversion. This isoelastic instantaneous utility also exhibits constant relative risk aversion and it rules out negative income.

A minimum consumption requirement for each period for the household was also set where minimum consumption is in corn output equivalents and tracks the household age composition. Table 5 shows the assumed minimum consumption. This assumption establishes the position of the household as a net seller in the market for corn and mango.

Table 5. Minimum Consumption (kg. in corn equivalents)

Household Member	Minimum Yearly Consumption
Household Head	50.0
Wife	37.5
Child (1-6 years old)	12.5
Child (7-10 years old)	25.0
Child (10-15 years old)	37.5

5.0. Results and Discussion

5.1. No Labor Constraints

The first to be discussed is the case where there are no labor market imperfections but there is full irreversibility. If there are no market imperfections then labor decisions are based on the market wage. In this case both uncertainty and irreversibility are exogenous. Exogenous irreversibility can happen for instance because of government regulations or policies that restrict or control fruit tree cutting in private lands. It could also happen possibly because of cultural beliefs that prevent trees from being cut, i.e. religious and cultural reasons.

The results for this scenario can be seen from Figures 1 and 2⁶. In figure 1 the farm size is 1 hectare, while in figure 2 the farm size is increased to 5 hectares. It is evident from the graphs that the whole household labor supply is used for off-farm work. This allocation decision makes small farmers net sellers of labor. Large farmers, on the other hand, are net sellers of labor at the early and late stages of the planning horizon, but are net buyers in the mid-periods. This corresponds to the nature of the assumption on households labor supply. In particular the U shaped pattern was assumed

⁶ The following symbols are used for all the graphs; FI – full irreversibility; PI – partial irreversibility (with restoration); NL – No labor constraints; LC – Labor constraints; L – farm size.

for household labor. The farm labor supply dips when the household is in the mid-stages of their life cycle. Therefore, at this stage the household can only maintain full cultivation of land by hiring labor.

Figure 1. Mango Land and Off Farm Labor Allocation (FI, NLC, L=1)

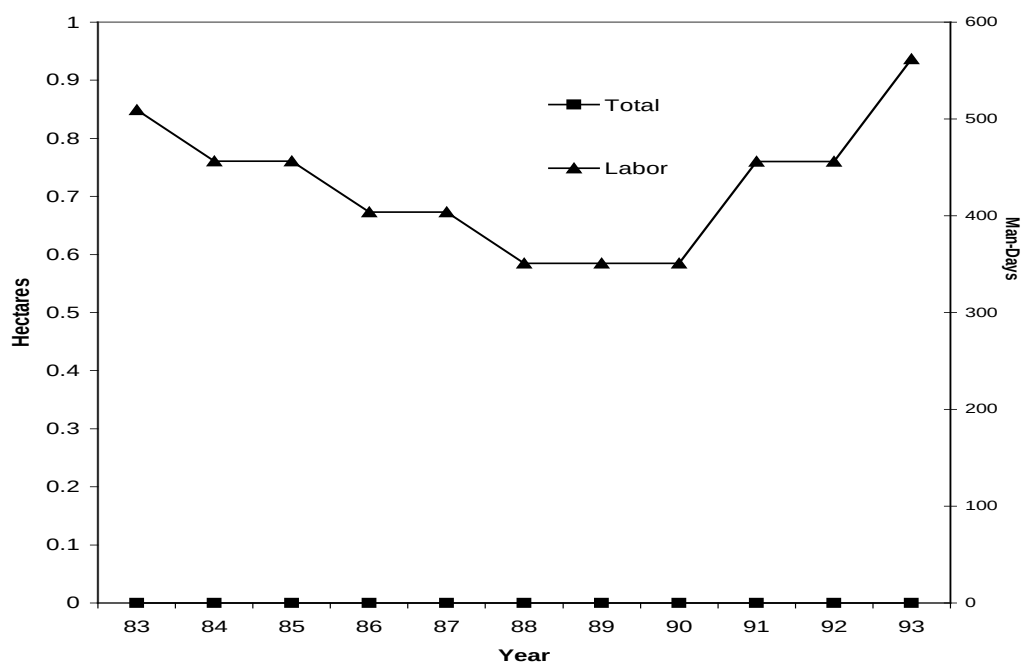
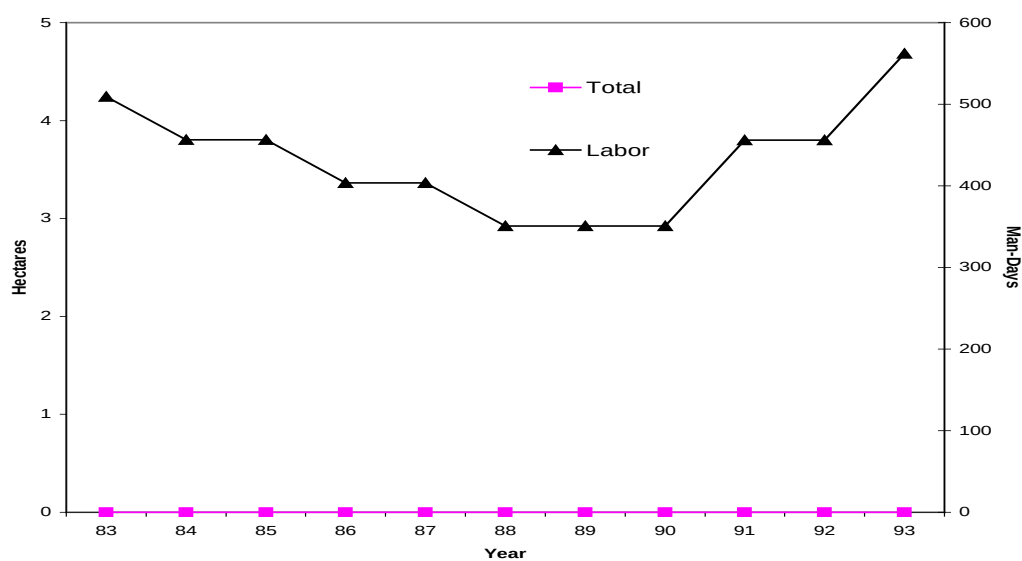


Figure 2. Mango Land and Off Farm Labor Allocation (FI, NLC, L=5)



Next, is the case where possible restoration of land back to annual cropping is allowed. The results are shown in Figure 3 and 4. When restoration is allowed and there are no market failures, the optimal wage, investment and labor allocation paths are the same as in the full irreversibility case. Again, farm labor requirements are met through hiring outside labor while household labor is used for non-farm work. These patterns also hold regardless of the farm size.

Figure 3. Mango Land and Off Farm Labor Allocation (PI, NLC, L=1)

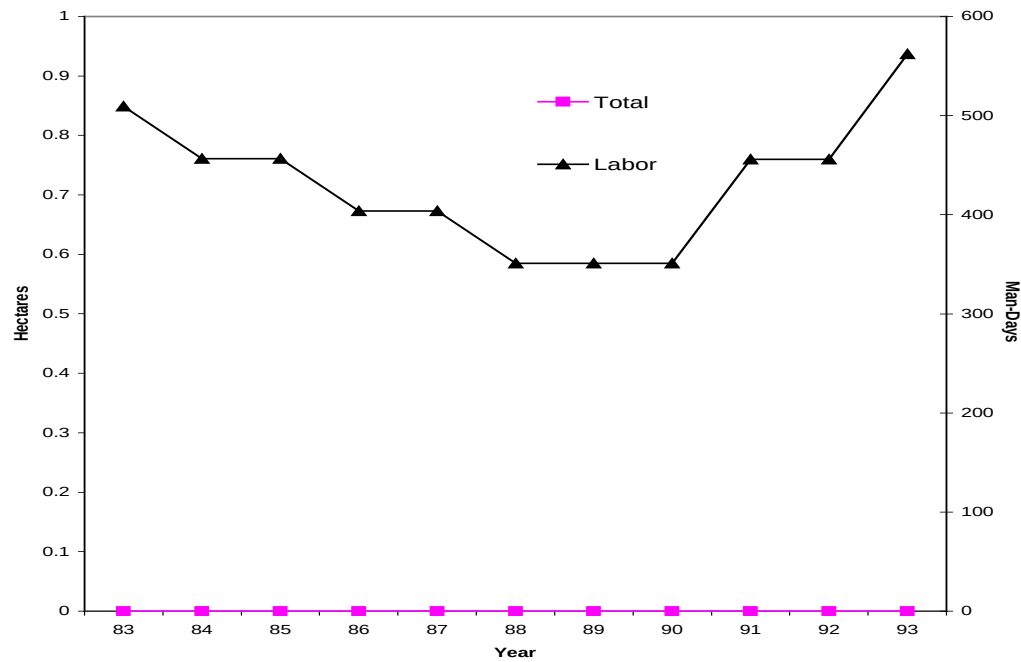
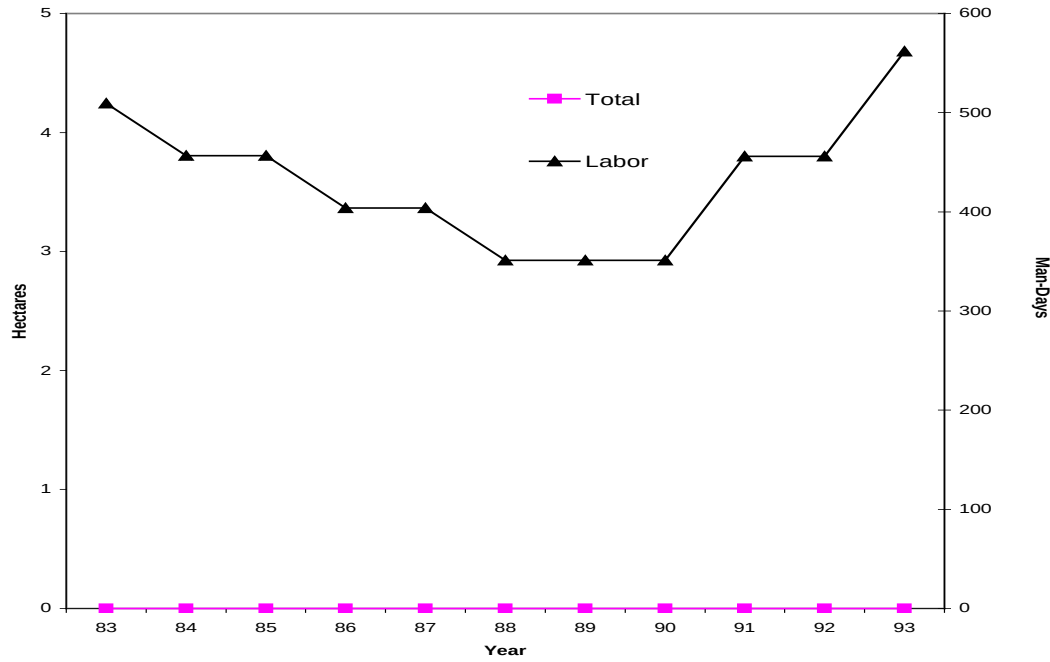


Figure 4. Mango Land and Off Farm Labor Allocation (PI, NLC, L=5)



These results show that when markets are complete, irreversibility and uncertainty do not have any distinguishable effect on tree planting. The often-observed delay in investment (in this case tree planting) is not observed even under full irreversibility simply because there is no investment in natural resource capital that is occurring. In fact, there was no diversification in land allocation portfolio and farmers tend to adopt monocropping, specifically full land allocation to annuals. The crux of this result might be the assumed low value for the risk aversion parameter θ . At this point a numerical experiment is conducted and one parameter is changed in the model. Specifically, θ was increased from 0.4 to 1.4. The results of this experiment are seen in Figures 5 and 6.

Figure 5. Mango Land Allocation (FI, NLC, L=1, theta=1.4)

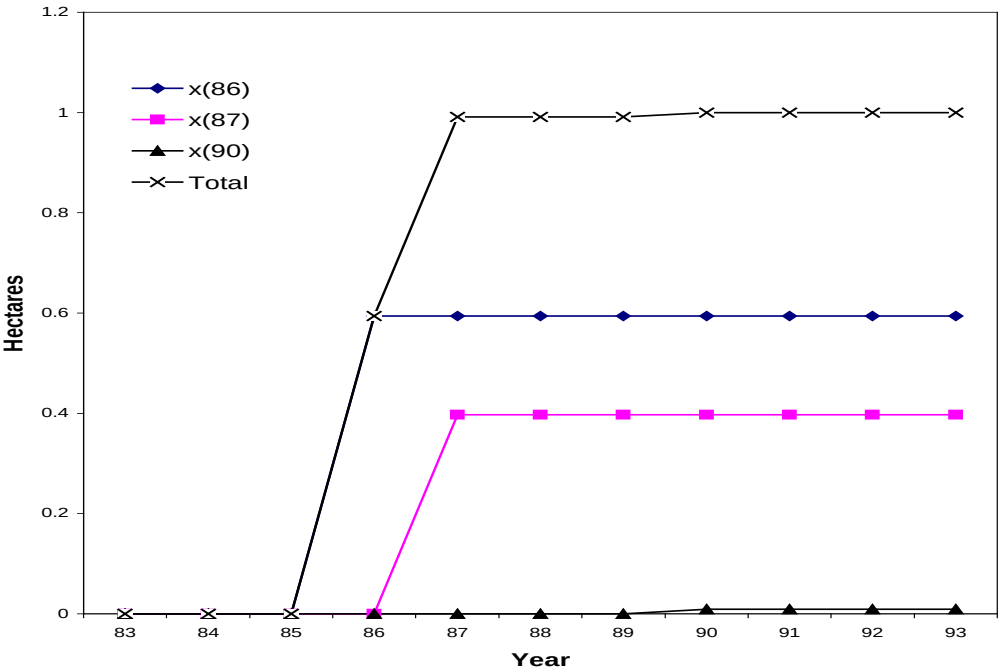


Figure 6. Mango Land and Off Farm Labor Allocation (FI, NLC, L=5, theta=1.4)

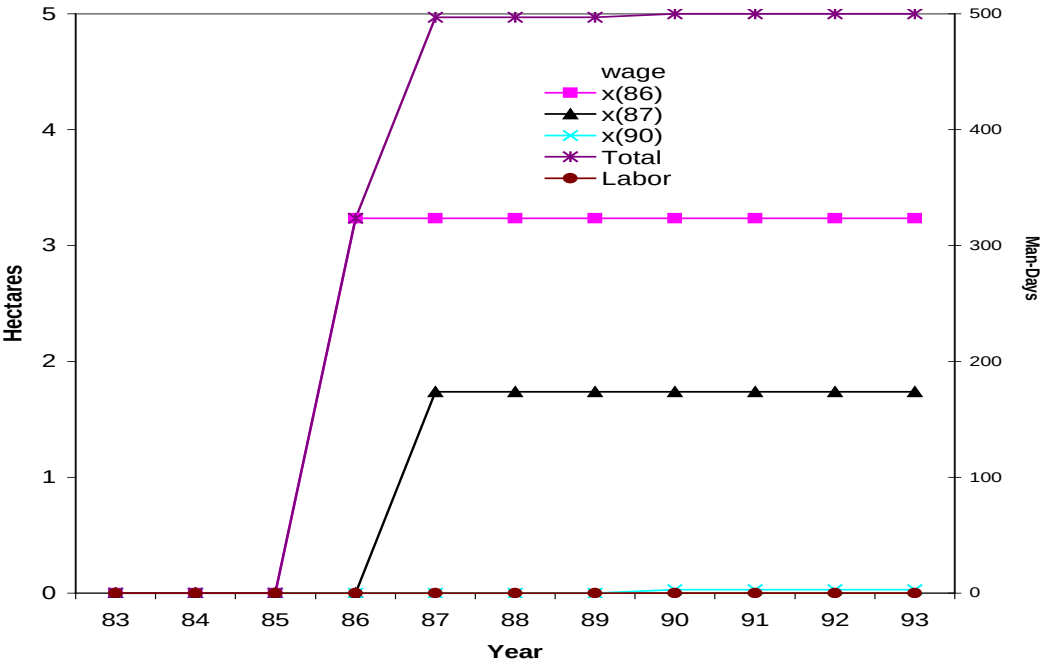


Figure 7. Mango Land Allocation (PI, NLC, L=1, theta=1.4)

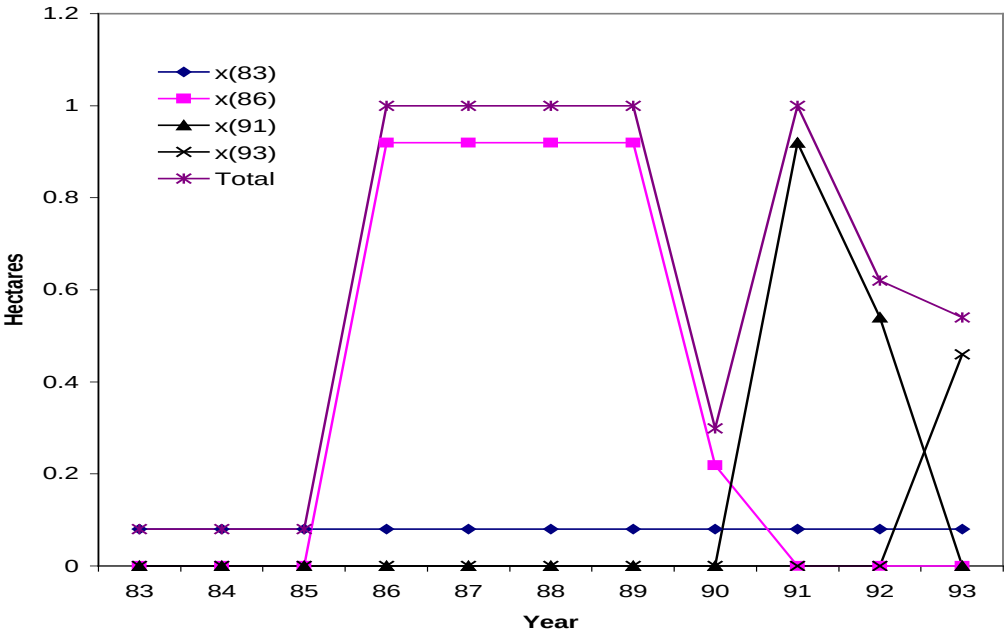
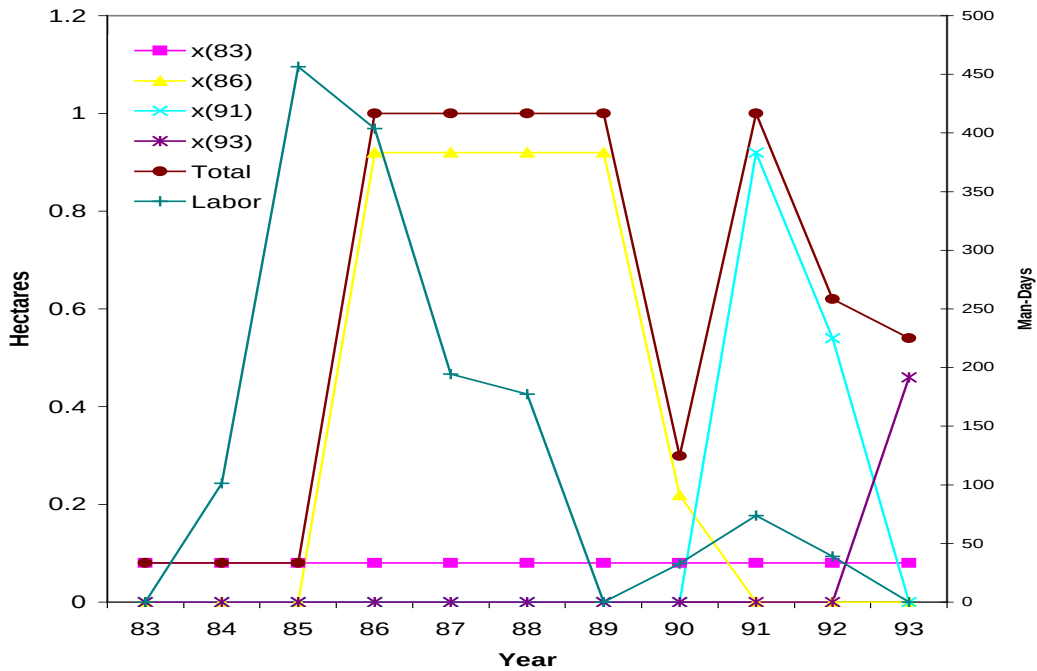


Figure 8. Mango Land and Off Farm Labor Allocation (PI, NLC, L=5, theta=1.4)



A difference in tree planting behavior can now be observed after assuming a higher value for θ . For the same optimal wage path, households move from periods of mono cropping to diversified land allocation. Mono cropping is now not exclusive to annuals. For households who face full irreversibility during the early stages land is exclusively for annuals but in the later stages it is shifted to tree farming. Outright diversification is not practiced but rather there is a delay in tree planting after increasing θ . When faced with irreversibility both small and large farmers will tend to plant at a later time (year 1986). Land allocation is then gradually increased until land is all allocated to mango production. This occurred towards the end of the planning horizon (year 1990).

When restoration of land is allowed, the diversification pattern is different (figures 7 and 8). There is early diversification followed by periods of switching between diversification and mono cropping where mono cropping is geared towards tree planting. This pattern of diversification means that there is no delay in tree investment. Thus, tree planting happens at the start of the planning horizon (year 1983). Full land allocation to mango happens early in the planning horizon but a pulsing pattern in mango tree planting can be observed. This pulsing pattern is a result of tree cutting activity. Younger trees are usually cut first since they have lower labor requirements and therefore lower cutting costs.

Diversification is one response of risk averse households to uncertainty. The increased tendency of farmers with higher risk aversion to plant trees can be understood by looking at the structural form of the instantaneous utility. The first order condition for the simulation model involves the marginal utility of income. With our assumption on the utility's structural form the marginal utility would be equal to $1/y^\theta$. The marginal utility of income increases as income decreases. This increase is higher when θ is higher. This means that farmers with higher risk aversion put a higher value on lower realizations of income than farmers with low risk aversion. Mango production is equal to zero for the first three years of cultivation hence, allocating land to trees yields a lower level of income than full allocation of land to corn during the early years of mango cultivation and therefore, a higher marginal utility. Thus, with the possibility of land restoration there is an early but gradual tree planting

The results have shown the importance of the role of risk aversion in the study of irreversibility, uncertainty and investment. In this study, a stylized discrete version of this problem has shown that under perfect markets, irreversibility may not be a factor affecting decisions if there is a low risk aversion among farmers. This is primarily due to the tendency of farmers to mono crop annuals and, therefore, avoiding the irreversible investment as represented by planting trees. This

also shows that there is a critical level of risk aversion that leads to diversification and therefore, the observance of the effects of irreversibility on tree planting.

5.2. Labor Constrained Households

The case where labor constraints are imposed will now be discussed and analyzed. Under this case the shadow wage results from the condition that household labor demand must equal the household labor supply. First to be analyzed is the standard discrete model of irreversibility and uncertainty with labor constraints. Figures 9, 10, 11, and 12 represent cases where irreversibility was exogenously imposed on the model (i.e. technical irreversibility). The main difference will be that the wage becomes endogenously determined within the model, but irreversibility is still imposed exogenously.

Second the case when irreversibility is determined by the ability of households to restore land back to annual cropping. The extent of restoration, and, therefore irreversibility is now determined by the labor constraint. Figures 13, 14, 15, and 16 represent cases where land restoration was allowed. This is the case where irreversibility is endogenous (i.e. economic irreversibility). In this model both the wages (and the process of uncertainty) and irreversibility are determined by the household's land and labor allocation.

5.2.1. Labor Constraint and Full Irreversibility

For the case of technical irreversibility, farmers with large farms tend to plant with considerable delay (figures 9 and 10). Initial tree planting occurred on their third year in farming (1985) and was followed by five more planting dates. The final cumulative land allocation to mangoes, however, was only approximately half of the whole farm. However, compared to the standard model wherein no labor constraints were imposed, planting occurred earlier. In the standard case, planting occurred during 1986 and by 1987 land was fully allocated to mango trees. When the labor constraint was imposed planting occurred in 1985 albeit at a lower tree density. The farm also remained diversified throughout the planning horizon. Thus, it can be seen that labor constraints lead to scale, timing, and rate effects.

Labor market participation is highest during the start of the planning horizon (30% of household labor supply) and at the end of the planning horizon (roughly 34%). The bulk of household labor is used in farming with the excess being used in off-farm labor market participation. Large farm owners did not hire outside labor and remained self-sufficient in terms of on-farm labor.

This indicates that households strategy under risk, irreversibility and labor market imperfections is one of diversification in both land and labor allocations. Labor diversification is a result of the inability of farmers to hire labor outside of the household. In the standard discrete model irreversibility, labor diversification was not prevalent. Households in this model hired outside labor and opted to participate full time in the labor market.

There is a difference in the allocation of farmers with smaller farms. This can be seen from figures 11 and 12. These farmers with smaller farms planted even later (year 1986) compared to farmers with larger farms. However, compared to large and small farmers of the standard discrete model, the timing is the same. The delay in planting is the same, i.e. farmers in both models planted in 1986. This suggests that irreversibility results into a delay in timing but labor constraints reduce the effects of full irreversibility and provides incentives for farmers to plant earlier.

The rate of planting is also different. For small farmers in the standard model there was a gradual planting scheme that eventually led to full land allocation to perennials. In contrast, when labor constraints were imposed farmers allocated land abruptly to perennials. In relative terms they had a higher planting intensity but less diversification occurred compared to the same type of farmers in the standard model.

They also participated more on off-farm labor markets albeit at a decreasing rate over time. Again, there was more varied labor allocation. Farmers, due to the inability to hire in labor, participated both on farm and off-farm work. Off-farm work was generally increasing through time.

The difference between the cumulative investment and intertemporal labor allocation patterns between small and larger farms can be attributed to the effect of labor constraints. Note that the labor constraint is more binding for larger farms since they have a larger land area to cultivate for the same household size. Thus, it is observed that small landowners are able to allocate their whole farms to mangoes. In sum, the imposition of labor constraints reduces the scale of tree planting and increases the extent of diversified farming when it is binding. It promotes early but slower rate of planting. When labor is in excess but there are constraints to hiring in labor, the timing of planting is not affected. There is a faster rate of planting but less diversification and cropping is geared towards perennials. Labor participation is also diversified between off-farm and farm work as a consequence of the inability to hire in labor. These differences can be attributed to the endogeneity of the wage process when there are labor market imperfections. The resulting wage process for each of the model will be analyzed later and linked to the observations in this section.

5.2.2. Labor Constraints with Restoration

The model wherein both irreversibility and the wages are determined endogenously within the system (figures 13 and 14) will now be analyzed and discussed. The timing of planting for large farmers is not affected by the modifications in the previous model. Farmers like those in the case where technical irreversibility was assumed, planted around 1985. This is still earlier compared to the standard model where restoration was allowed (partial irreversibility). The notable difference between these two labor constraint models is in the scale of tree planting. Although the rate and timing are the same, the scale or intensity of planting per year and the cumulative tree density at the end of the planning horizon is lower for the case of endogenous irreversibility. Planting peaked at around 2.3 hectares for these farmers while for farmers faced with exogenous irreversibility it was around 3 hectares. The final cumulative tree density for farmers in the current model is only 1.4 hectares while in the previous model it was 3 hectares.

Cutting occurred around 1992. Again younger trees were cut earlier as they entail lower labor requirements. During this period, corn prices increased to the same level during the start of the planning horizon. Note that corn prices have been going down erratically from 1983 to 1991. In total, approximately 1.6 hectares of trees were cut. However, in 1992, 0.986 hectares of trees were planted. Since, this was lower than the cut area the final cumulative land allocation was lower than previous years. The final land allocation is also lower compared to the allocation of labor-constrained households who faced full irreversibility.

In terms of off-farm work, the labor demand is the same compared to the previous full irreversibility problem during the early periods. Labor demand for off-farm work was, however, lower at the end of the planning horizon. This is due to the fact that the household restored some land back to annuals and at the same time planted new trees. These simultaneous activities resulted to higher labor demand on the farm in 1991 considering that labor requirements for perennials are highest during the planting stages or the first three years of mango cultivation. Note also that the labor demand like that in the case of technical irreversibility tracks the household's labor supply pattern. Again the labor supply is determined by the stage of the households in its lifecycle.

Figure 9. Mango Land and Off Farm Labor Allocation (FI, LC, L=5)

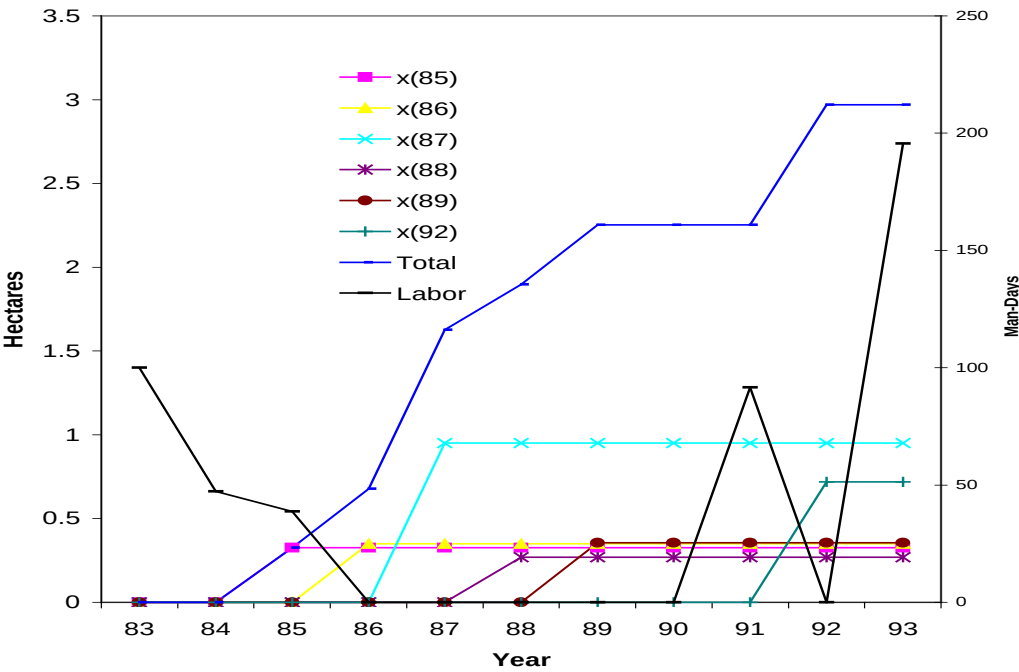


Figure 10. Mango Land Allocation (FI, LC, L=5)

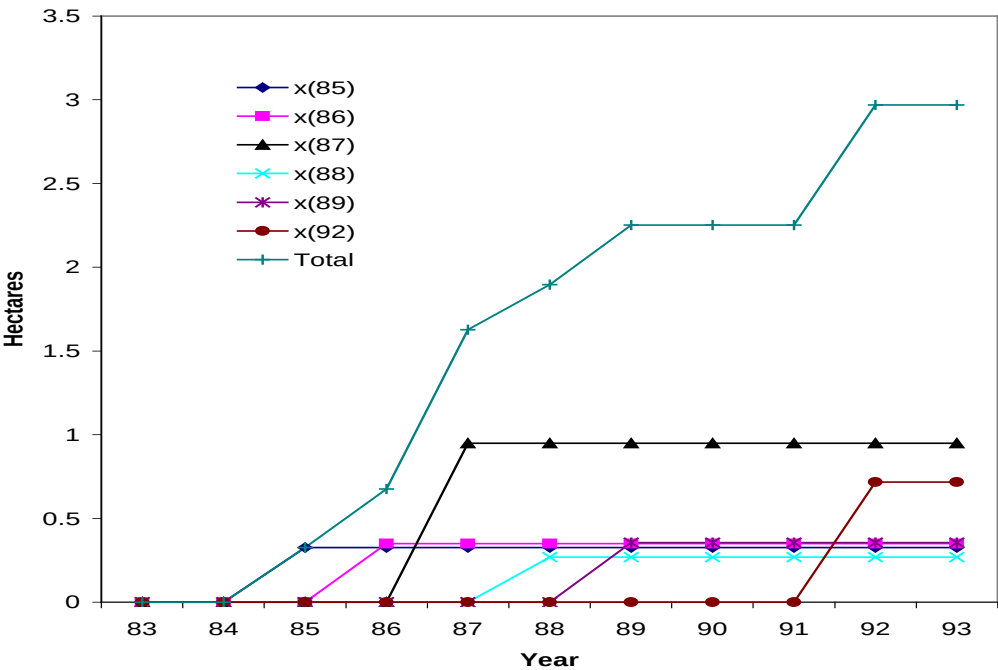


Figure 11. Mango Land and Off Farm Labor Allocation (FI, LC, L=1)

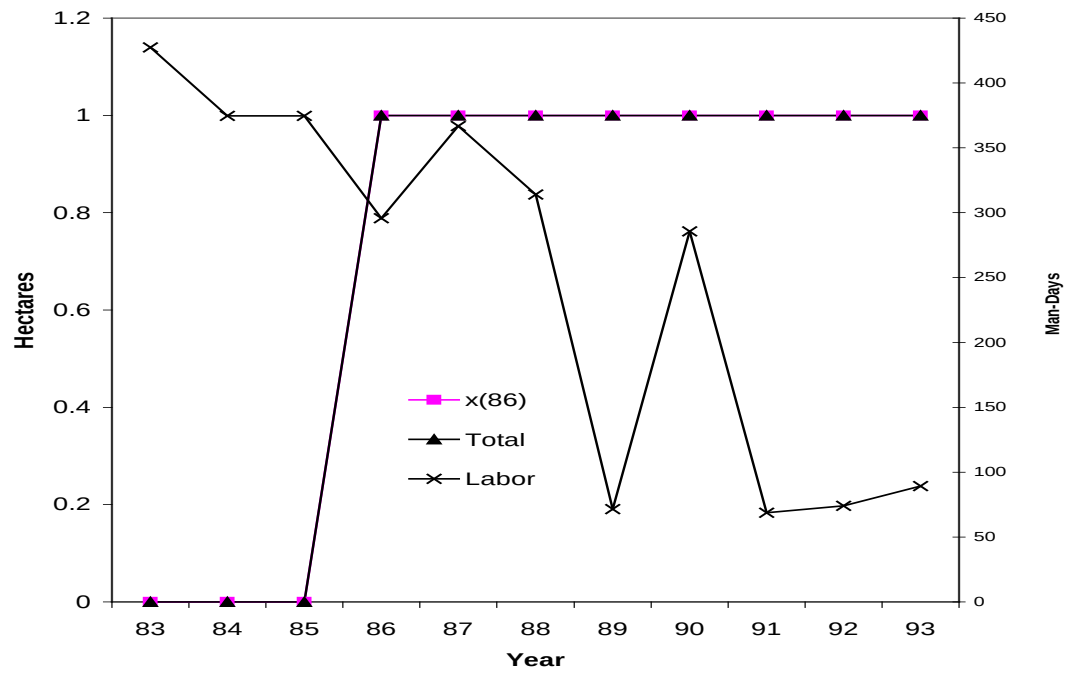


Figure 12. Mango Land Allocation (FI, LC, L=1)

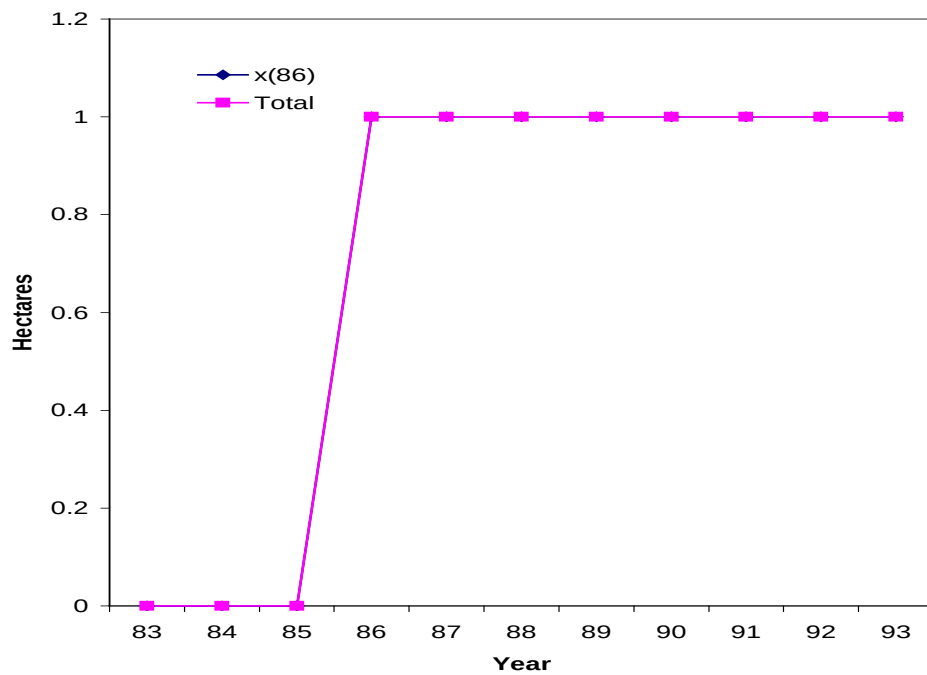


Figure 13. Mango Land and Off Farm Labor Allocation (PI, LC, L=5)

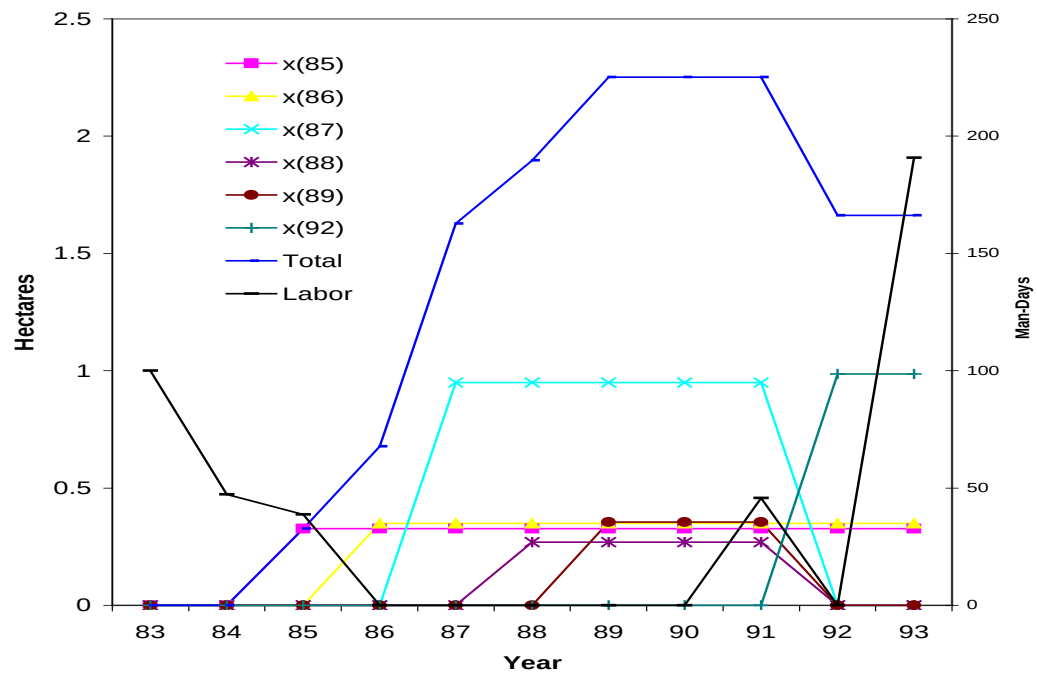
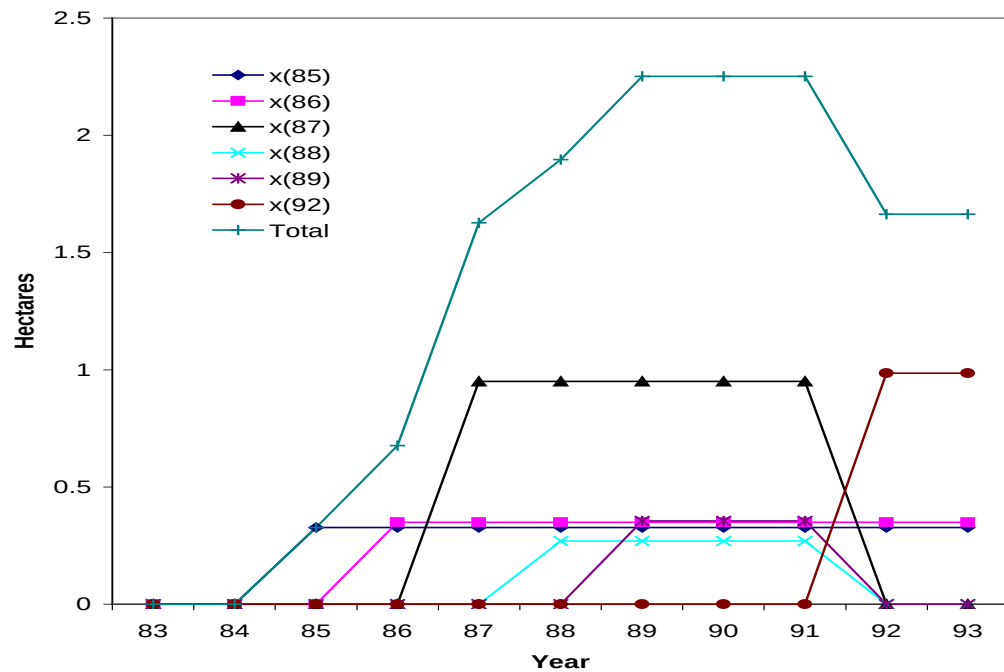


Figure 14. Mango Land Allocation (PI, LC, L=5)



For households who owned smaller farms (figures 15 and 16), again the date of planting was not different from that of the case where technical irreversibility and labor constraints were assumed. The initial dates of planting were both done during 1986. However, the rates of planting were different. As seen earlier, under exogenous irreversibility, households with small farms abruptly shifted from annual cropping to tree farming. In the present model, planting intensity was slower allowing for periods where mixed planting of annuals and perennials or agroforestry systems existed. There was also no scale effect. The final cumulative amount of land planted with trees was the same. Thus, only the rate of planting was affected. However, unlike in the previous model, occurrence of tree rotation was observed. Specifically, unlike large farm owners these small farm owners clear cut their existing trees in 1992 and replaced them with newer trees. For the large farm owners they only cut younger trees and kept the older trees that required more labor but probably provided providing more production. Complete rotation did not occur among these large farm owners unlike the small farm owners. Thus, when labor constraints are more binding farmers will tend to “age” some of their existing trees.

The behavior of the households under full irreversibility with labor constraints and under endogenous irreversibility was generally the same. However, the ability to restore land back to annuals had a profound effect on the intertemporal welfare of the household. If the maximized sum of discounted expected utility is used as a welfare measure, then farmers with the ability to restore land would have higher welfare levels. The welfare level (in monetary terms) of households who own 5 hectares of land (and thus, were more labor constrained) and faced full irreversibility is 14,822.70 pesos while those with the ability to restore amount to 17, 682.79 pesos. This is approximately a 16% reduction. The same magnitude was observed for small landowners. The irreversibility effect, therefore accounts, for roughly a 16% reduction in welfare levels of labor-constrained households.

Figure 15. Mango Land and Off Farm Labor Allocation (PI, LC, L=1)

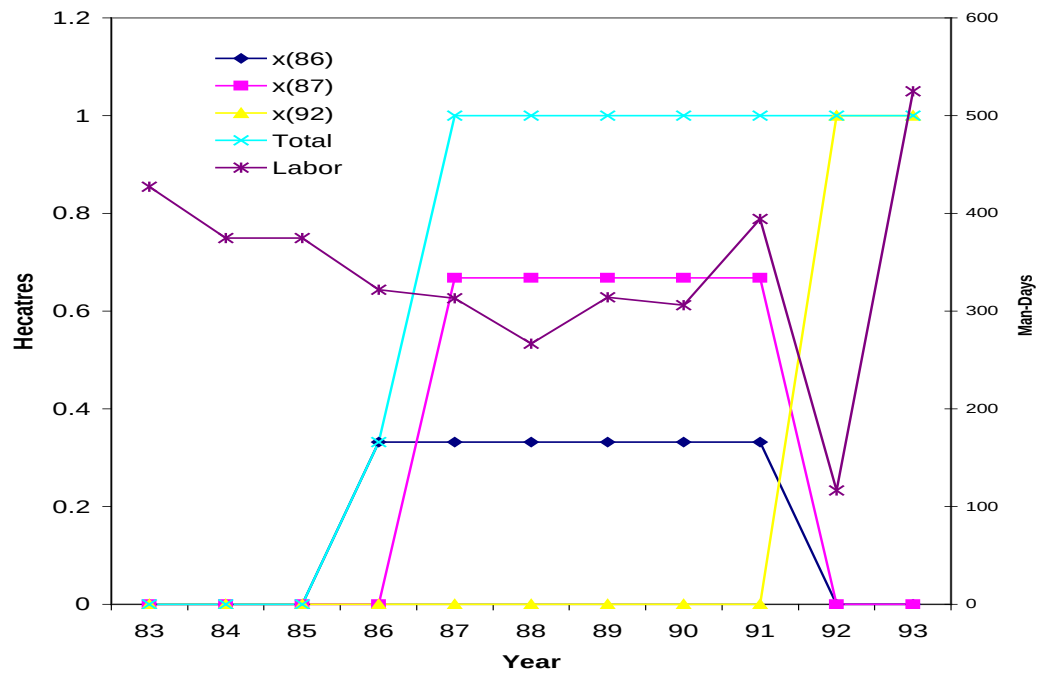
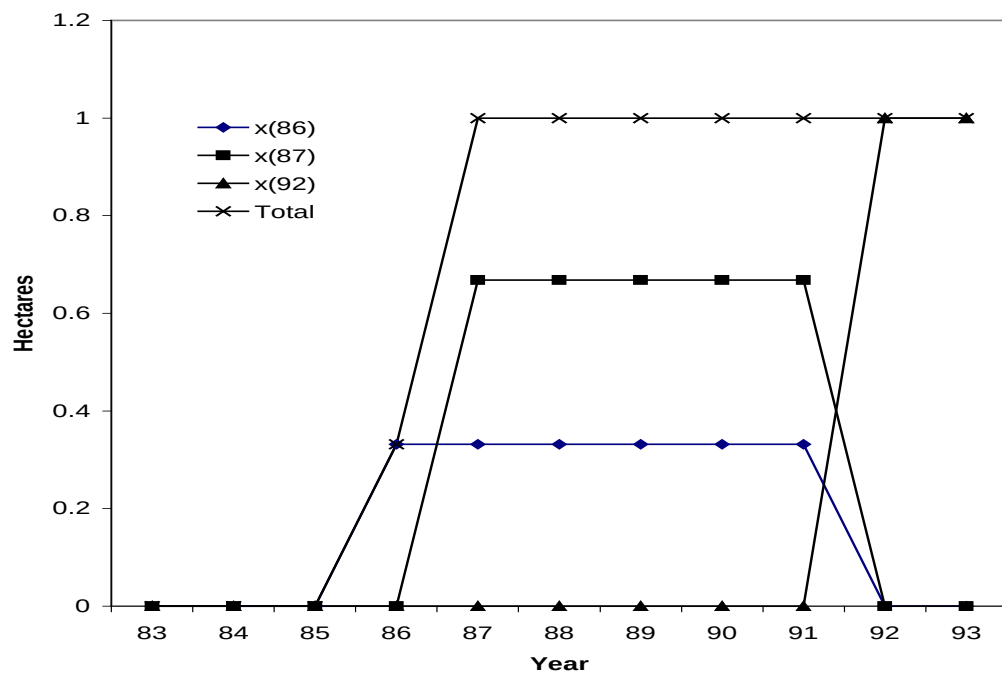


Figure 16. Mango Land Allocation (PI, LC, L=1)



5.3. Shadow Wage Process

Previously Table 1 showed the market and the shadow wage process that clears the internal household labor market. It can be seen that the shadow base wage for the labor-constrained household is higher than the market base wage. In terms of the essential parameters of the binomial wage process, the drift and instantaneous variance are higher for labor-constrained household under full irreversibility compared to the case wherein restoration is possible. For small farm owners there was a reduction in the drift rate by 25% and 58% for the instantaneous variance. In contrast, large farm owners had a more modest reduction in their parameters. In particular, there was only a 40% reduction in variance and a 4% reduction in the drift rate. This also points to the interesting finding that, allowing for the possibility of restoration, reduces the instantaneous variance for both types of farmers.

When compared to the wage parameters of the market wage, there is a difference between large and small farm owners. Regardless of whether the farmer is faced with exogenous irreversibility or endogenous irreversibility, under labor constraints the small farmer would have a higher instantaneous variance and a higher drift rate compared to the market wage process. On the other hand, large farm owners had a lower instantaneous variance but a slightly higher drift rate compared to the market wage process. Since household with larger farms would have more labor deficits, it can be concluded that regardless of the type or source of irreversibility the presence of market imperfections leads to a lower instantaneous variance but higher drift rates.

The difference in the instantaneous variance may also explain why there was a longer delay in planting for small landed households compared to large farm owners. In related studies on investment under irreversibility and uncertainty it has been shown that increasing the instantaneous variance increases the hurdle rate for investment (Dixit and Pindyck, 1994). Thus, a longer delay in the timing of investment is observed. This is exactly what was observed earlier for smaller farms. In the model's set-up, the instantaneous variance enters the equation for the value of the up and down trend. A higher instantaneous variance increases the value of the up trend and decreases the value of the downtrend, which eventually affects the timing of investment.

For large farmers, the ability to restore land to annuals (i.e. endogenous irreversibility) lowers both the instantaneous variance and the drift rates. The same pattern is seen for small farmers. This seems to point out that, regardless of the extent of labor constraint, the ability to restore land even partially reduces the instantaneous variance and the drift rates. It can be concluded, therefore, that there is a difference between the effects of irreversibility and the labor constraint. Irreversibility

under imperfect markets in general increases the instantaneous variance and drift rates regardless of the extent of irreversibility. On the other hand, regardless of the type of irreversibility, households who have excess labor, but cannot hire in labor had higher instantaneous variance and drift rates compared to labor deficit households.

At this point it is useful to expound on the role of the drift parameter in the binomial wage model. This parameter enters the equation for the probability of an increase in the wage in the future. A higher drift parameter means a higher probability of a wage increase in future periods and would affect the rate or speed of tree planting. For small labor constrained farmers, it was noted, that not only was there a delay in planting, but it occurred only at one time with full land allocation. This is likely explained by the high drift parameter. A strategy of gradual land allocation to perennials would increase the likelihood of having higher wage costs. The wage cost is determined by the labor allocation and wage realizations. By assumption, mango labor requirements are highest during the first three years of cultivation. Thus, when the realized wage is low and the drift rate is high, it is more prudent to allocate the optimum amount of land at first planting than gradually building up to the desired level of investment and face a higher likelihood of higher wage costs in the future. Since labor constraint for farm work for small farmers is less binding, it is possible to allocate the whole land area immediately to tree crops. In comparison, a gradual tree planting strategy was observed with the case where there was no labor constraint in which the decision was based on the market wage. The market wage had a zero drift rate.

6.0. Conclusion and Policy Implications

This paper was motivated by the observation and reality that trees in general are easy to plant and hard to uproot in the later stages of their biological growth. The difference between the initial investment in this natural resource capital and the cost of adjustment in the future results into a condition of irreversibility. The main objective is to see the implications of this condition on the tree planting (investment) and cutting (disinvestments) decisions of the household. Irreversibility has consequences for farmers because it reduces the flexibility of their land allocation systems, and therefore, reduces their ability to cope with risk ex-post.

Irreversibility can come from a variety of sources. First, it can be imposed exogenously on a farmer through policy constraints that hinder cutting of trees. This is exogenous irreversibility and is the same as the standard model of irreversibility and uncertainty. On the other hand, irreversibility may be due to economic constraints that hinder disinvestments in the future. This is economic or

endogenous irreversibility and is the main interest of the study. In particular, the emphasis of this paper is exploring the role of labor market imperfections or labor constraints and its interaction with the biophysical nature of trees and how the resulting endogenous irreversibility affect the planting and cutting behavior of upland farmers.

A discrete time continuous state simulation model was used to observe the tree-planting behavior of households. A discrete time analog of the continuous time geometric Brownian process was used for the model. This structure of the model has several advantages; first, it helps avoid dealing with more difficult partial differential equations, which has been associated with continuous time versions of this problem, and second, it permits the use of LaGrange optimization, which is a simpler solution method than stochastic dynamic optimization. Lastly, the structure saves programming costs since it can be easily programmed using optimization software such as GAMS.

Three models were constructed to explore possible theoretical propositions that can be obtained from a model of irreversibility and market imperfections. The first model was the discrete time version of the standard irreversibility model. The second model was the standard irreversibility model with labor market imperfections. In particular, irreversibility was still imposed exogenously but a condition was added wherein the household is not able to hire outside labor. Finally, the last model is the case wherein labor market imperfections were the source of irreversibility. Under this model the extent of irreversibility is anchored on the extent of labor constraint. Furthermore, in each of these models two types of farmers were assumed; those with large farms and those who owned small farms. The difference lies in their position when labor market supply constraints are imposed. Large farm owners would have labor deficits while small landowners would have excess labor.

The three models were compared in terms of the resulting patterns in land and labor allocation to trees. The following theoretical proposition were obtained from the exercise:

1. Under the standard model, the effects of irreversibility, whether partial or full irreversibility can only be seen if risk aversion is high enough. In other words, there is a critical level of risk aversion before irreversibility becomes a factor.
2. Irreversibility under imperfect markets increases the instantaneous variance and drift rates regardless whether households have labor deficits or surpluses.
3. Under market imperfections, the status of the labor supply has effects on the instantaneous variance and drift rates that are independent of the source of irreversibility. In particular, labor deficits (large farm owners) leads to lower instantaneous variance but higher drift rates. On the other hand, farmers with farm labor surplus have higher drift rates and instantaneous variance compared to labor deficit farmers and the perfect market case.

In terms of the planting patterns the following observations were noted:

1. Under the standard model with perfect markets, the possibility of restoration results into early planting but gradual planting culminating to mono cropping of trees.
2. Under market imperfections, regardless of the source of irreversibility, labor deficit households will plant earlier compared to the standard model. In contrast, regardless of the source of irreversibility households with labor surplus will exhibit no difference in the timing of planting compared to the standard model. Compared to labor deficit farmers, labor surplus farmers will plant at a later date.
3. Under market imperfections, labor deficit households would plant at a slower rate regardless of the source of irreversibility. In contrast households with labor surplus will plant faster under a situation when irreversibility is imposed exogenously than when restoration is permitted but constrained by labor.
4. Under market imperfections, there is a scale effect for labor deficit households depending on the source of irreversibility. Economic irreversibility results into a lower final cumulative amount of land allocated to trees. This shows the possibility of investment “overshooting” when early investment mistakes are made but are not possible to correct. No scale effects are evident for labor surplus households.
5. Under market imperfection, household with labor surplus may adopt a full rotation scheme when irreversibility is endogenous. In contrast, labor deficit households adopt only partial rotation. Furthermore, younger trees are the ones that will be cut leaving some trees to “age”. Furthermore, small farm owners or labor surplus households plant more tree per hectare.
6. Endogenous irreversibility increases the households welfare by as much as 16%.

These theoretical propositions obtained from a stylized simulation model needs to be tested using actual field data. Despite of this deficiency, the strength of the modeling approach to decision-making is the ability to incorporate variables and assumptions into the simulation process as well as the ability to vary these various elements and assumptions to create varying conditions and scenarios. In this specific study, one implication is that given a heterogeneous group of households in these communities, government programs must promote a portfolio of incentives to enhance perennial tree crop introduction into these upland systems. For example, for upland communities near urban or industrialized centers which could absorb excess labor from these households, and where farm size are small and land tenure is secure, tree crop planting is most likely to occur based on the outputs of the simulation model. All that is needed here would be for government to put in place mechanisms for making planting materials available to small farmers.

For upland communities experiencing labor market imperfections which is most likely to occur in remote areas far or with low accessibility to urban centers, assistance must be provided to households facing labor constraints. It is also a challenge to provide farm household with increased capacity for restoration and flexibility by buffering risks. This could come in the form of low interest loans, which is administered by providing farm inputs in kind rather than cash to encourage tree-crop planting. The organization of labor groups could also be a good institutional intervention. This type of local level institution may assume the role of the missing labor market.

The previous discussions points to a need for a more tailored approach to agroforestry intervention. Another way to interpret the policy implication of this study is that there should be more targeting of potential agroforestry participants. For upland communities who have access to a diverse labor market, the program participants that should be targeted are those who have fewer assets or the poorer cross-section of the community. They are more likely to plant trees even under the threat of irreversibility and uncertainty. On the other hand, in communities that are far from the urban center, households that have larger farms and less farm manpower should be encouraged to participate.

Whether the tailored or targeting approach is used, the wider implication of the study is clear. Agroforestry intervention should do away with a programmatic approach, which has the tendency to implement blanket policies. A more suitable strategy is the project approach, which provides more flexibility in implementation with various portfolios of appropriate options considering the diversity of conditions in the uplands. This approach results into a higher probability of success for government and private sector interventions in promoting tree planting considering the conditions in the uplands.

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